A Unified Picture with Neutrino As a Central Feature [*]

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Abstract

In the first part of this talk it is discussed why observed neutrino oscillations (which suggest the existence of right-handed neutrinos with certain Dirac and Majorana masses) seem to select out the route to higher unification based on the symmetry SU(4)-color. This in turn selects out the effective symmetry in 4D near the GUT/string scale to be either SO(10) or minimally $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$. The same conclusion is reached by the likely need for leptogenesis as the means for baryogenesis and also by the success of certain fermion mass-relations including $m_b(M_{GUT}) \approx m_\tau$, together with $m(\nu^\tau)_{Dirac} \approx m_{top}(M_{GUT})$.

In the second part, an attempt is made to provide a unified picture of a set of diverse phenomena based on an effective G(224) symmetry or SO(10), possessing supersymmetry. The phenomena in question include: (a) fermion masses and mixings, (b) neutrino oscillations, (c) CP non-conservation, (d) flavor violations in quark and lepton sectors, as well as (e) baryogenesis via leptogenesis. Including SM and SUSY contributions, the latter being sub-dominant, the framework correctly accounts for Δm_K, Δm_{B_d}, $S(B_d \to J/\psi K_s)$ and ϵ_K, and predicts $S(B_d \to \phi K_s)$ to be in the range +(0.65–0.73), close to the SM-prediction. It also quite plausibly accounts for the observed baryon excess $Y_B \approx 10^{-10}$. Furthermore the model predicts enhanced rates for $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\mu N \to eN$ and also measurable electric dipole moment for the neutron. Expectations arising within the same framework for proton decay are summarized at the end. It is stressed that the two notable missing pieces of the framework are supersymmetry and proton decay. While search for supersymmetry at the LHC is eagerly awaited, that for proton decay will need the building of a megaton-size detector.

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1 Introduction

Since the discoveries (confirmations) of the atmospheric [1] and solar neutrino oscillations [2,3], the neutrinos have emerged as being among the most effective probes into the nature of higher unification. Although almost the feeblest of all the entities of nature, simply by virtue of their tiny masses, they seem to possess a subtle clue to some of the deepest laws of nature pertaining to the unification-scale and (even more important) to the nature of the unification-symmetry. In this sense the neutrinos provide us with a rare window to view physics at truly short distances. As we will see, these turn out to be as short as about 10^{-30} cm. In addition,it appears most likely that the origin of their tiny masses may be at the root of the origin of matter-antimatter asymmetry in the early universe. In short, the neutrinos may be crucial to shedding light not only on unification but also on our own origin!

The main purpose of this talk would be two-fold. First I discuss in the next section the issue of the choice of the effective symmetry in 4D. Here, I explain why (a) observed neutrino oscillations, (b) the likely need for leptogenesis as the means for baryogenesis [4,5], and (c) the success of certain fermion mass relations, *together*, seem to select out the route to higher unification based on the symmetry SU(4)-color [6, 7]. The effective symmetry near the GUT/string scale in 4D should thus be either SO(10) [8], or minimally $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ [7], as opposed to other alternatives. The second part of my talk is based on recent works on fermion masses and neutrino oscillations [9], and CP and flavor violations [10,11], all treated within a promising SO(10)/G(224) framework. The purpose of this second part is to present a *unified description* of a set of diverse phenomena, including:

- Fermion masses and mixings

- Neutrino oscillations

- CP non-conservation

- Flavor violations (in quark *and* lepton sectors),

- Baryogenesis via leptogenesis, and

- Proton Decay.

As it turns out, the neutrino plays a central role in arriving at this unified picture. My goal here will be to exhibit that the first five phenomena hang together neatly, in accord with observations, within *a single predictive framework*, based on an effective symmetry in 4D which is either SO(10) or G(224).

As we will see, the predictions of the framework not only account for many of the features of the five phenomena listed above (including the smallness of V_{cb}, the near maximality of Θ_{23}^{ν}, $m_b(m_b)$, $\Delta m^2(\nu_2-\nu_3)$, ϵ_K, $S(B_d \to J/\psi K_S)$, baryon asymmetry Y_B, and more), but also include features involving CP and flavor violations (such as edm, the asymmetry parameter $S(B_d \to \phi K_S)$ and $\mu \to e\gamma$) which can clearly test the framework on many fronts.

To set the background for this discussion I first remark in the next section on the choice of the effective symmetry in 4D and the need for SU(4)-color. In this connection, I also clarify the historical origin of some of the concepts that are common to both G(224) and SO(10) and are now crucial to an understanding of neutrino masses and implementing baryogenesis via leptogenesis. In the following section, I briefly review the SO(10)/G(224)-framework proposed in Ref. [9] for considerations of fermion masses and neutrino oscillations, and in the subsequent sections discuss the issues of CP and flavor violations [10, 11] as well as baryogenesis via leptogenesis [5], within the same framework. Expectations for proton decay are noted at the end.

2 On the choice of the Effective Symmetry in 4D: The need for SU(4)-color

The idea of grand unification was motivated [6,7,13] by the desire to explain (a) the observed quantum numbers of the members of a family, and (b) quantization of electric charge on the one hand, and simultaneously to achieve (c) unification of quarks and leptons and (d) a unity of the basic forces on the other hand. While these four, together with the observed gauge coupling unification [14] , still provide the strongest support – on aesthetic and empirical grounds– in favor of grand unification, they leave open the question of the choice of the effective symmetry G in 4D near the GUT scale which achieves these four goals.

For instance, should the symmetry group G be of rank 4, that is SU(5) [13], which is devoid of SU(4)-color? Or, should G possess SU(4)-color and thus minimally be SO(10)

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of rank 5, or even E_6 [15] of rank 6? Or, should G be a string-derived semi-simple group G(224) $\subset$ SO(10), still of rank 5? Or, should G be $[SU(3)]^3 \subset E_6$, of rank 6, but devoid of SU(4)-color?

An answer to these questions that helps select out the effective symmetry G in 4D is provided, however, if together with the four features (a)–(d) listed above, one folds in the following three:

(e) Neutrino oscillations

(f) The likely need for leptogenesis as the means for baryogenesis, and

(g) The success of certain fermion mass relations noted below

One can argue [12] that the last three features, together with the first four listed above, clearly suggest that the standard model symmetry very likely emerges, near the GUT-scale $M_U \sim 2 \times 10^{16}$ GeV, from the spontaneous breaking of a higher gauge symmetry G that should possess the symmetry SU(4)-color [7]. The relevant symmetry in 4D could then maximally be SO(10) (possibly even E_6 [15]) or minimally the symmetry G(224); either one of these symmetries may be viewed to have emerged in 4D [16, 17] from a string/M theory near the string scale $M_{st} \gtrsim M_{GUT}$ [1]. The theory thus described should of course possess weak scale supersymmetry so as to avoid unnatural fine tuning in Higgs mass *and* to ensure gauge coupling unification.

To see the need for having SU(4)-color as a component of the higher gauge symmetry, it is useful to recall the family-multiplet structure of G(224), which is retained by SO(10) as well. The symmetry G(224), subject to left-right discrete symmetry which is natural to G(224), organizes members of a family into a single left-right self-conjugate multiplet $(\mathrm{F}_L^e \bigoplus \mathrm{F}_R^e)$

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[1]The relative advantage of an effective string-derived SO(10) over a G(224)-solution and vice versa have been discussed in detail in [12]. Briefly speaking, for the case of a string derived G(224)-solution, coupling unification being valid near the string scale, one needs to assume that the string scale is not far above the GUT scale ($M_{st} \approx (2-3)M_{GUT}$, say) to explain observed gauge coupling unification. While such a possibility can well arise in the string theory context [18], for an SO(10)-solution, coupling unification at the GUT-scale is ensured regardless of the gap between string and GUT-scales. The advantage of a G(224)-solution over an SO(10) solution is, however, that doublet-triplet splitting (DTS) can emerge naturally for the former in 4D through the process of string compactification (see Ref. [17]), while for an SO(10)-solution this feature is yet to be realized. As we will see, SO(10) and G(224) share many common advantages, aesthetic and practical, in particular as regards an understanding of fermion masses, neutrino oscillations and baryogenesis via leptogenesis; but they can be distinguished empirically through phenomena involving CP and flavor violations as well as proton decay.

given by [7]:

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \tag{1}$$

The multiplets F^e_L and F^e_R are left-right conjugates of each other transforming respectively as **(2, 1, 4)** and **(1, 2, 4)** of G(224); likewise for the muon and the tau families. Note that *each family of G(224), subject to left-right symmetry, must contain sixteen two-component objects* as opposed to fifteen for SU(5) [13] or the standard model. While the symmetries $SU(2)_{L,R} \subset G(224)$ treat each column of $F^e_{L,R}$ as doublets, the symmetry SU(4)-color unifies quarks and leptons by treating each row of F^e_L and F^e_R as a quartet. Thus SU(4)-color treats the left and right-handed neutrinos (ν^e_L and ν^e_R) as the *fourth color-partners* of the left and right-handed up quarks (u_L and u_R) respectively. Here in lies the distinctive feature of SU(4)-color. *It necessitates the existence of the RH neutrino (ν^e_R) on par with that of the RH up quark (u_R) by relating them through a gauge symmetry transformation;* and likewise for the mu and the tau families. As we will see, this in turn leads to some very desirable fermion mass relations for the third family that help distinguish it from alternative symmetries. *An accompanying characteristic of SU(4)-color is that it also introduces $B - L$ as a local symmetry* [7]. This in turn plays a crucial role in protecting the Majorana masses of the right-handed neutrinos from acquiring Planck-scale values.

In anticipation of sections. 3, 4 and 7 where some of the statements made below will become clear, I may now state the following. The need for SU(4)-color (mentioned above) arises because it provides the following desirable features:

(1) RH neutrino (ν^i_R) as an essential member of each family

Needed to implement the seesaw mechanism and leptogenesis (see Secs. 3 and 7).

(2) $B - L$ as a local symmetry

Needed to protect ν'_Rs from acquiring Planck scale masses and to set $M(\nu^i_R) \propto M_{B-L} \sim M_{GUT}$.

(3) Two simple mass relations for the 3rd family:

(a) $m(\nu^\tau_{Dirac}) \approx m_{top}(M_{GUT})$

Needed for success of seesaw (see section 3).

(b) $m_b(M_{GUT}) \approx m_\tau$

Empirically successful.

These three ingredients ((1), (2) and (3a)), together with the SUSY unification-scale, are in-

deed crucial (see sections 3 and 4) to an understanding of the neutrino masses via the seesaw mechanism [19]. The first two ingredients are important also for implementing baryogenesis via leptogenesis [4,5] (see section 7). *Hence the need for having SU(4)-color as a component of the unification symmetry which provides all four ingredients.*

By contrast SU(5), devoid of SU(4)-color, does not provide the ingredients of (1), (2) and (3a) (though it does provide (3b)); hence it does not have a natural setting for understanding neutrino masses and implementing baryogenesis via leptogenesis (see discussion in section 4 and especially footnote 2). Symmetries like $G(2213) = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^c$ [20] and $[SU(3)]^3$ [21] provide (1) and (2) but neither (3a) nor (3b), while flipped $SU(5)' \times U(1)'$ [22] provides (1), (2) and (3a) but not (3b). In summary, *the need for the combination of the four ingredients (1), (2), (3a) and (3b) seems to select out the route to higher unification based on SU(4)-color*, and thereby as mentioned above an effective symmetry like G(224) or SO(10) being operative in 4D near the string scale.

At this point, an intimate link between $SU(4)$-color and the left-right symmetric gauge structure $SU(2)_L \times SU(2)_R$ is worth noting. Assuming that $SU(4)^c$ is gauged and demanding an explanation of quantization of electric charge lead one to gauge minimally the left-right symmetric flavor symmetry $SU(2)_L \times SU(2)_R$ (rather than $SU(2)_L \times U(1)_{I_{3R}}$). The resulting minimal gauge symmetry that contains $SU(4)$-color and explains quantization of electric charge is then $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ [7]. With SU(4)-color being vectorial, such a symmetry structure (as also G(2213) which is a subgroup of G(224)) in turn naturally suggests the attractive idea that L–R discrete symmetry and thus parity (i.e. $F_L \leftrightarrow F_R$, $W_L \leftrightarrow W_R$ with $g_L^{(0)} = g_R^{(0)}$) is preserved at a basic level and is broken only spontaneously [23]. In other words, observed parity violation is only a low-energy phenomenon which should disappear at sufficiently high energies. *We thus see that the concepts of SU(4)-color and left-right symmetry are intimately inter-twined, through the requirement of quantization of electric charge.*

A Historical Note: Advantages of G(224)

As a historical note, it is worth noting that the symmetry $SU(4)$-color, and thereby the three desirable features listed above, were introduced into the literature, as a step towards higher unification, through the minimal symmetry G(224) [7], rather than through SO(10) [8]. The

symmetry G(224) (supplemented by L–R discrete symmetry which is natural to G(224)) in turn brought a host of desirable features. Including those mentioned above they are:

(i) Unification of all sixteen members of a family within *one* left-right self-conjugate multiplet, with a neat explanation of their quantum numbers;

(ii) Quantization of electric charge;

(iii) Quark-lepton unification through $SU(4)$-color;

(iv) Conservation of parity at a fundamental level [23];

(v) RH neutrino as a compelling member of each family;

(vi) $B - L$ as a local symmetry; and

(vii) The rationale for the now successful mass-relations (3a) and (3b).

These seven features constitute the *hallmark* of G(224). Any simple or semi-simple group that contains G(224) as a subgroup would of course naturally possess these features. So does therefore SO(10) which is the smallest simple group containing G(224). Thus, as alluded to above, *all the attractive features of SO(10), which distinguish it from SU(5) and are now needed to understand neutrino masses and baryogenesis via leptogenesis, were in fact introduced through the symmetry G(224) [7], long before the SO(10) papers appeared [8].* These in particular include the features (i) as well as (iii)–(vii). SO(10) of course preserved these features for reasons stated above; it even preserved the family multiplet structure of G(224) without needing additional fermions (unlike E_6) in that the L–R conjugate **16-plet** ($= F_L \bigoplus F_R$) of G(224) precisely corresponds to the spinorial **16** ($= F_L \bigoplus (F_R)^c$) of SO(10). Furthermore, with SU(4)-color being vectorial, G(224) is anomaly-free; so also is SO(10).

SO(10) brought of course one added and desirable feature relative to G(224)– that is manifest coupling unification. Again, as a historical note, it is worth mentioning that the idea of coupling unification was initiated in [6] and was first manifested explicitly within a minimal model through the suggestion of SU(5) in [13].

As mentioned before, believing in string unification, either G(224) or SO(10) may be viewed to have its origin in a still higher gauge symmetry (like E_8) in 10D. To realize the existence of the right-handed neutrinos, $B - L$ as a local symmetry and the fermion mass-relations (3a), which are needed for understanding neutrino masses and implementing baryogenesis via leptogenesis, I have argued that one needs $SU(4)^c$ as a component of the effective symmetry in 4D, and therefore minimally G(224) (or even G(214)) or maximally

perhaps SO(10) in 4D near the string scale. The relative advantages of G(224) over SO(10) and vice versa as 4D symmetries in addressing the issues of doublet-triplet splitting on the one hand and gauge coupling unification on the other hand have been discussed in Ref. [12] and briefly noted in footnote 1.

In the following sections I discuss how either one of these symmetries G(224) or SO(10) link together fermion masses, neutrino oscillations, CP and flavor violations and leptogenesis. As we will see, while G(224) and SO(10) lead to essentially identical results for fermion masses and neutrino oscillations, which are discussed in the next two sections, they can be distinguished by processes involving CP and/or flavor violations, which are discussed in sections 5 and 6, and proton decay, discussed in section 8.

3 Seesaw and SUSY Unification with SU(4)-color

The idea of the seesaw mechanism [19] is simply this. In a theory with RH neutrinos as an essential member of each family, and with spontaneous breaking of $B - L$ and I_{3R} at a high scale (M_{B-L}), both already inherent in [7], the RH neutrinos can and generically will acquire a superheavy Majorana mass ($M(\nu_R) \sim M_{B-L}$) that violates lepton number and $B - L$ by two units. Combining this with the Dirac mass of the neutrino ($m(\nu_{\text{Dirac}})$), which arises through electroweak symmetry breaking, one would then obtain a mass for the LH neutrino given by

$$m(\nu_L) \approx m(\nu_{\text{Dirac}})^2 / M(\nu_R) \tag{2}$$

which would be naturally super-light because $M(\nu_R)$ is naturally superheavy. This then provided a *simple but compelling reason* for the lightness of the known neutrinos. In turn it took away the major burden that faced the ideas of $SU(4)$-color and left-right symmetry from the beginning. In this sense, the seesaw mechanism was indeed the missing piece that was needed to be found for consistency of the ideas of $SU(4)$-color and left-right symmetry.

In turn, of course, the seesaw mechanism needs the ideas of $SU(4)$-color and SUSY grand unification so that it may be quantitatively useful. Because the former provides (a) the RH neutrino as a compelling feature (crucial to seesaw), and (b) the Dirac mass for the tau neutrino accurately in terms of the top quark mass (cf. feature (3a)), while the latter provides the superheavy Majorana mass of the ν_R^τ in terms of the SUSY unification scale (see

Sec. 4). Both these masses enter crucially into the seesaw formula and end up giving the *right mass-scale* for the atmospheric neutrino oscillation as observed. To be specific, $SU(4)$-color yields: $m(\nu^\tau_{\text{Dirac}}) \approx m_{\text{top}}(M_U) \approx 120$ GeV; and the SUSY unification scale, together with the protection provided by $B - L$ that forbids Planck-scale contributions to the Majorana mass of ν^τ_R, naturally yields: $M(\nu^\tau_R) \sim M^2_{GUT}/M \sim 4 \times 10^{14}$ GeV(1/2–2), where $M \sim 10^{18}$ GeV (1/2–2) [cf. Sec. 4]. The seesaw formula (without 2-3 family mixing) then yields:

$$m(\nu^3_L) \approx (120 \text{ GeV})^2/(4 \times 10^{14} \text{ GeV}(1/2\text{--}2)) \approx (1/28 \text{ eV})(1/2\text{--}2)) \tag{3}$$

With hierarchical pattern for fermion mass-matrices (see Sec. 4), one necessarily obtains $m(\nu^2_L) \ll m(\nu^3_L)$ (see section 4), and thus $\sqrt{\Delta m^2_{23}} \approx m(\nu^3_L) \sim 1/28$ eV(1/2–2). This is just the right magnitude to go with the mass scale observed at SuperK [1]!

Without an underlying reason as above for at least the approximate values of these two vastly differing mass-scales — $m(\nu^\tau_{\text{Dirac}})$ and $M(\nu^\tau_R)$ — the seesaw mechanism by itself would have no clue, quantitatively, to the mass of the LH neutrino. In fact it would yield a rather arbitrary value for $m(\nu^\tau_L)$, which could vary quite easily by more than 10 orders of magnitude either way around the observed mass scale. This would in fact be true if one introduces the RH neutrinos as a singlet of the SM or of SU(5).[2]

In short, the seesaw mechanism needs the ideas of SUSY unification and $SU(4)$-color, and of course vice-versa; *together* they provide an understanding of neutrino masses as observed.

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[2]To see this, consider for simplicity just the third family. Without $SU(4)$-color, even if a RH two-component fermion N (the analogue of ν_R) is introduced by hand as a *singlet* of the gauge symmetry of the SM or $SU(5)$, *such an N by no means should be regarded as a member of the third family, because it is not linked by a gauge transformation to the other fermions in the third family.* Thus its Dirac mass term given by $m(\nu^\tau_{\text{Dirac}})[\bar{\nu}^\tau_L N + h.c.]$ is completely arbitrary, except for being bounded from above by the electroweak scale ~ 200 GeV. In fact a priori (within the SM or $SU(5)$) it can well vary from say 1 GeV (or even 1 MeV) to 100 GeV. Using Eq. (2), this would give a variation in $m_{(\nu_L)}$ by at least four orders of magnitude if the Majorana mass $M(N)$ of N is held fixed. Furthermore, N being a singlet of the SM as well as of $SU(5)$, the Majorana mass $M(N)$, unprotected by $B - L$, could well be as high as the Planck or the string scale (10^{18}–10^{17} GeV), and as low as say 1 TeV; this would introduce a further arbitrariness by fourteen orders of magnitude in $m(\nu_L)$. Such arbitrariness both in the Dirac and in the Majorana masses, is drastically reduced, however, once ν_R is related to the other fermions in the family by an $SU(4)$-color gauge transformation and a SUSY unification is assumed.

Schematically, one thus finds:

$$
\boxed{\begin{array}{c} \text{SUSY UNIFICATION} \\ \text{WITH } SU(4)\text{-COLOR} \end{array}} \quad \oplus \quad \boxed{\text{SEESAW}}
$$

$$\Downarrow$$

$$m(\nu_L^3) \quad \sim \quad 1/10 \text{ eV}.$$

(4)

In summary, as noted in section 2, the agreement of the expected $\sqrt{\Delta m_{23}^2}$ with the observed SuperK value clearly seems to favor the idea of the seesaw and select out the route to higher unification based on supersymmetry and SU(4)-color, as opposed to other alternatives.

I will return to a more quantitative discussion of the mass scale and the angle associated with the atmospheric neutrino oscillations in Sec. 4.

4 Fermion Masses and Neutrino Oscillations in G(224)/SO(10): A Review of the BPW framework

Following Ref. [9], I now present a simple and predictive pattern for fermion mass-matrices based on $SO(10)$ or the $G(224)$-symmetry.[3] One can obtain such a mass mass-matrix for the fermions by utilizing only the minimal Higgs system that is used also to break the gauge symmetry $SO(10)$ to $SU(3)^c \times U(1)_{em}$. It consists of the set:

$$H_{\text{minimal}} = \left\{ \mathbf{45_H}, \mathbf{16_H}, \overline{\mathbf{16_H}}, \mathbf{10_H} \right\} \tag{5}$$

Of these, the VEV of $\langle \mathbf{45_H} \rangle \sim M_U$ breaks $SO(10)$ in the B-L direction to $G(2213) = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^c$, and those of $\langle \mathbf{16_H} \rangle = \langle \overline{\mathbf{16_H}} \rangle \sim M_U$ along $\langle \tilde{\bar{\nu}}_{RH} \rangle$ and $\langle \tilde{\nu}_{RH} \rangle$ break $G(2213)$ into the SM symmetry $G(213)$ at the unification-scale M_U. Now $G(213)$ breaks at the electroweak scale by the VEVs of $\langle \mathbf{10_H} \rangle$ and of the EW doublet in $\langle \mathbf{16_H} \rangle$ to $SU(3)^c \times U(1)_{em}$.

Before discussing fermion masses and mixings, I should comment briefly on the use of the minimal Higgs system noted above as opposed to large-dimensional tensorial multiplets of SO(10) including $(126_H, \overline{126_H})$, 210_H and possibly 120_H, which have been used widely in

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[3]I will present the Higgs system for $SO(10)$. The discussion would remain essentially unaltered if one uses the corresponding $G(224)$-submultiplets instead.

the literature [24] to break SO(10) to the SM symmetry and give masses to the fermions. We have preferred to use the low-dimensional Higgs multiplets (45_H, 16_H and $\overline{16_H}$) rather than the large dimensional ones like (126_H, $\overline{126_H}$, 210_H and possibly 120_H) in part because these latter tend to give too large GUT-scale threshold corrections to $\alpha_3(m_Z)$ from split sub-multiplets (typically exceeding 15–20% with either sign), which would render observed gauge coupling unification fortuitous (see Appendix D of Ref. [9] for a discussion on this point). By contrast, with the low-dimensional multiplets (45_H, 16_H and $\overline{16_H}$), the threshold corrections to $\alpha_3(m_Z)$ are tamed and are found, for a large range of the relevant parameters, to have the right sign and magnitude (nearly -5 to -8%) so as to account naturally for the observed gauge coupling unification.

Another disadvantage of 126_H, which contributes to EW symmetry breaking through its (2, 2, 15) component of G(224), is that it gives $B - L$ dependent contribution to family-diagonal fermion masses. Such a contribution, barring adjustment of parameters against the contribution of $\langle 10_H \rangle$, could in general make the success of the relation $m_b(GUT) \approx m_\tau$ fortuitous. By contrast, the latter relation emerges as a robust prediction of the minimal Higgs system ($45_H, 16_H, \overline{16_H}$ and 10_H), subject to a hierarchical pattern, because the only $(B - L)$ dependent contribution in this case can come effectively through $\langle 10_H \rangle \langle 45_H \rangle / M$ which is family-antisymmetric and cannot contribute to diagonal entries (see below) [9].

One other issue involves the question of achieving doublet-triplet splitting by a natural mechanism as opposed to that of fine tuning, and incorporating the associated GUT-scale threshold correction to $\alpha_3(m_Z)$. For the case of (45_H, 16_H, $\overline{16_H}$ and 10_H), there exists a simple mechanism which achieves the desired splitting naturally with the introduction of an extra $10'_H$ [25], and the effect of this splitting on GUT-scale threshold correction to $\alpha_3(m_Z)$ has been evaluated in [9] to conform with natural coupling unification on the one hand and the limit on proton lifetime on the other hand. To the best of my knowledge, an analogous study for the system involving (126_H, $\overline{126_H}$) has not been carried out as yet.

Balancing against these advantages of the minimal Higgs system, the large-dimensional system ($126_H, \overline{126_H}, 210_H$ and possibly 120_H) has an advantage over the minimal system, because 126 and $\overline{126}$ break $B - L$ by two units and thus automatically preserve the familiar R-parity = $(-1)^{3(B-L)+2S}$. By contrast, 16 and $\overline{16}$ break $B - L$ by one unit and thereby break the familiar R-parity. This difference is, however, not really significant, because for the

minimal system one can still define consistently a matter-parity (i.e. $16_i \to -16_i$, $16_H \to$ 16_H, $\overline{16_H} \to \overline{16_H}$, $45_H \to 45_H$, $10_H \to 10_H$), which serves the desired purpose by allowing all the desired interactions but forbidding the dangerous $d = 4$ proton decay operators and yielding stable LSP to serve as CDM. Given the net advantages of the minimal Higgs system H_{minimal}, as noted above, I will proceed to present the results of [9] which uses this system.[4]

The 3×3 Dirac mass matrices for the four sectors (u, d, l, ν) proposed in Ref. [9] were motivated in part by the notion that flavor symmetries [26] are responsible for the hierarchy among the elements of these matrices (i.e., for "33" $\gg$ "23" $\sim$ "32" $\gg$ "22" $\gg$ "12" $\gg$ "11", etc.), and in part by the group theory of SO(10)/G(224), relevant to a minimal Higgs system. Up to minor variants [27], they are as follows [5,6]:

$$
M_u = \begin{bmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \zeta_{22}^u & \sigma + \epsilon \\ 0 & \sigma - \epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \qquad M_d = \begin{bmatrix} 0 & \eta' + \epsilon' & 0 \\ \eta' - \epsilon' & \zeta_{22}^d & \eta + \epsilon \\ 0 & \eta - \epsilon & 1 \end{bmatrix} \mathcal{M}_d^0
$$

$$
M_\nu^D = \begin{bmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & \zeta_{22}^u & \sigma - 3\epsilon \\ 0 & \sigma + 3\epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \quad M_l = \begin{bmatrix} 0 & \eta' - 3\epsilon' & 0 \\ \eta' + 3\epsilon' & \zeta_{22}^d & \eta - 3\epsilon \\ 0 & \eta + 3\epsilon & 1 \end{bmatrix} \mathcal{M}_d^0
$$

(6)

These matrices are defined in the gauge basis and are multiplied by $\bar{\Psi}_L$ on left and Ψ_R on right. For instance, the row and column indices of M_u are given by $(\bar{u}_L, \bar{c}_L, \bar{t}_L)$ and (u_R, c_R, t_R) respectively. Note the group-theoretic up-down and quark-lepton correlations: the same σ occurs in M_u and M_ν^D, and the same η occurs in M_d and M_l. It will become clear that the ϵ and ϵ' entries are proportional to $B - L$ and are antisymmetric in the family space

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[4]Personally I feel, however, that it would be important to explore thoroughly the theoretical and phenomenological consequences of both the minimal and the large-dimensional Higgs systems involving issues such as doublet-triplet splitting, GUT-scale threshold corrections to gauge couplings, CP and flavor violations and proton decay. The aim would be to look for avenues by which the two systems can be distinguished experimentally.

[5]A somewhat analogous scheme based on low dimensional $SO(10)$ Higgs multiplets, has been proposed by C. Albright and S. Barr [AB] [28], who, however use two pairs of $(16_H, \overline{16_H})$, while BPW use only one. One major difference between the work of AB and that of BPW [9] (stemming from the use of two pairs of $(16_H, \overline{16_H})$ by AB compared to one by BPW) is that the AB model introduces the so-called "lop-sided" pattern in which some of the "23" and "32" elements are even greater than the "33" element; in the BPW model on the other hand, the pattern is consistently hierarchical with individual "23" and "32" elements (like η, ϵ and σ) being much smaller in magnitude than the "33" element of 1. It turns out that this difference leads to a characteristically different explanation for the large (maximal) $\nu_\mu - \nu_\tau$ oscillation angle in the two models, and in particular to a much more enhanced rate for $\mu \to e\gamma$ in the AB model compared to that in the BPW model (see Sec. 7).

[6]An alternate SO(10)-based pattern differing from BPW and AB models is proposed in [29].

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(as shown above). Thus, the same ϵ and ϵ' occur in both (M_u and M_d) and also in (M_ν^D and M_l), but $\epsilon \to -3\epsilon$ and $\epsilon' \to -3\epsilon'$ as $q \to l$. Such correlations result in enormous reduction of parameters and thus in increased predictiveness. Such a pattern for the mass-matrices can be obtained, using a minimal Higgs system $\mathbf{45}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H$ and $\mathbf{10}_H$ and a singlet S of SO(10), through effective couplings as follows [30] (see Ref. [9] and [12] for details):

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Yuk}} \;=\; & h_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H + [h_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(S/M) \\
& +\; a_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^p + g_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^q] \\
& +\; \left[h_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{10}_H(S/M)^2 + g_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+1}\right] \\
& +\; \left[g_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+2} + a_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^{p+2}\right] \;\;.(7)
\end{aligned}
$$

Typically we expect M', M'' and M to be of order M_{string} or (possibly) of order M_{GUT} [31]. The VEV's of $\langle \mathbf{45}_H \rangle$ (along $B-L$), $\langle \mathbf{16}_H \rangle = \langle \overline{\mathbf{16}}_H \rangle$ (along standard model singlet sneutrino-like component) and of the SO(10)-singlet $\langle S \rangle$ are of the GUT-scale, while those of $\mathbf{10}_H$ and of the down type $\mathrm{SU}(2)_L$-doublet component in $\mathbf{16}_H$ (denoted by $\mathbf{16}_H^d$) are of the electroweak scale [9,32]. Depending upon whether $M'(M'') \sim M_{\mathrm{GUT}}$ or M_{string} (see [31]), the exponent $p(q)$ is either one or zero [33].

The entries 1 and σ arise respectively from h_{33} and h_{23} couplings, while $\hat{\eta} \equiv \eta - \sigma$ and η' arise respectively from g_{23} and g_{12}-couplings. The ($B-L$)-dependent antisymmetric entries ϵ and ϵ' arise respectively from the a_{23} and a_{12} couplings. [Effectively, with $\langle \mathbf{45}_H \rangle \propto B-L$, the product $\mathbf{10}_H \times \mathbf{45}_H$ contributes as a $\mathbf{120}$, whose coupling is family-antisymmetric.] The relatively small entry ζ_{22}^u arises from the h_{22}-coupling, while ζ_{22}^d arises from the joint contributions of h_{22} and g_{22}-couplings.

Such a hierarchical form of the mass-matrices, with h_{33}-term being dominant, is attributed in part to a U(1)-flavor gauge symmetry [10,12] that distinguishes between the three families and introduces powers of $\langle S \rangle/M \sim 1/10$, and in part to higher dimensional operators involving for example $\langle \mathbf{45}_H \rangle/M'$ or $\langle \mathbf{16}_H \rangle/M''$, which are suppressed by $M_{\mathrm{GUT}}/M_{\mathrm{string}} \sim 1/10$, if M' and/or $M'' \sim M_{\mathrm{string}}$.

The right-handed neutrino masses arise from the effective couplings of the form [34]:

$$
\mathcal{L}_{\mathrm{Maj}} = f_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H/M \tag{8}
$$

where the f_{ij}'s include appropriate powers of $\langle S \rangle/M$. The hierarchical form of the Majorana

mass-matrix for the RH neutrinos is [9]:

$$M_R^\nu = \begin{bmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{bmatrix} M_R \tag{9}$$

Following flavor charge assignments (see [12]), we have $1 \gg y \gg z \gg x$. The magnitude of M_R is estimated by putting $f_{33} \approx 1$ and $\langle \overline{\mathbf{16}}_H \rangle \approx M_{GUT} \approx 2 \times 10^{16}$ GeV. We expect that the effective scale M of Eq. (8) should lie between $M_{string} \approx 4 \times 10^{17}$ GeV and $(M_{Pl})_{reduced} \approx 2 \times 10^{18}$ GeV. Thus we take $M \approx 10^{18}$ GeV (1/2–2) [9,12]. We then get the Majorana mass of the heaviest RH neutrino to be given by $M_3 \approx M_R = f_{33} \langle \overline{\mathbf{16}}_H \rangle^2 / M \approx (4 \times 10^{14} \text{ GeV})(1/2\text{–}2)$.

Ignoring possible phases in the parameters and thus the source of CP violation for a moment, and also setting $\zeta_{22}^d = \zeta_{22}^u = 0$, as was done in Ref. [9], the parameters $(\sigma, \eta, \epsilon, \epsilon', \eta', \mathcal{M}_u^0, \text{ and } \mathcal{M}_d^0)$ can be determined by using, for example, $m_t^{\mathrm{phys}} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $m_s(1 \text{ GeV}) = 110 - 116$ MeV, $m_u(1 \text{ GeV}) = 6$ MeV, and the observed masses of e, μ, and τ as inputs. One is thus led, *for this CP conserving case*, to the following fit for the parameters, and the associated predictions [9]:

$$\sigma \approx 0.110, \quad \eta \approx 0.151, \quad \epsilon \approx -0.095, \quad |\eta'| \approx 4.4 \times 10^{-3},$$
$$\epsilon' \approx 2 \times 10^{-4}, \quad \mathcal{M}_u^0 \approx m_t(M_X) \approx 100 \text{ GeV}, \quad \mathcal{M}_d^0 \approx m_\tau(M_X) \approx 1.1 \text{ GeV}. \tag{10}$$

These output parameters remain stable to within 10% corresponding to small variations ($\lesssim 10\%$) in the input parameters of m_t, m_c, m_s, and m_u. These in turn lead to the following

predictions for the quarks and light neutrinos [9], [12]:

$$m_b(m_b) \approx (4.7\text{–}4.9) \text{ GeV},$$
$$\sqrt{\Delta m_{23}^2} \approx m(\nu_3) \approx (1/24 \text{ eV})(1/2\text{–}2),$$
$$V_{cb} \approx \left| \sqrt{\frac{m_s}{m_b} \left| \frac{\eta+\epsilon}{\eta-\epsilon} \right|} - \sqrt{\frac{m_c}{m_t} \left| \frac{\sigma+\epsilon}{\sigma-\epsilon} \right|} \right|$$
$$\approx 0.044,$$

$$\begin{cases} \theta_{\nu_\mu\nu_\tau}^{\text{osc}} \approx \left| \sqrt{\frac{m_\mu}{m_\tau} \left| \frac{\eta-3\epsilon}{\eta+3\epsilon} \right|^{1/2}} + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \right| \\ \approx |0.437 + (0.378 \pm 0.03)| \text{ (for } \frac{m(\nu_2)}{m(\nu_3)} \approx 1/6\text{)}, \\ \text{Thus, } \sin^2 2\theta_{\nu_\mu\nu_\tau}^{\text{osc}} \approx 0.993, \end{cases}$$

$$V_{us} \approx \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \right| \approx 0.20,$$
$$\left| \frac{V_{ub}}{V_{cb}} \right| \approx \sqrt{\frac{m_u}{m_c}} \approx 0.07,$$
$$m_d(1 \text{ GeV}) \approx 8 \text{ MeV}. \tag{11}$$

It has been noted [12, 35] that small non-seesaw contribution to $\nu_L^e \nu_L^\mu$ mass term ($\sim$ few $\times 10^{-3}$ eV) which can arise through higher dimensional operators in accord with flavor symmetry, but which have been ignored in the analysis given above, can lead quite plausibly to large $\nu_e - \nu_\mu$ oscillation angle in accord with the LMA MSW solution for the solar neutrino problem. Including the seesaw contribution obtained by combining M_ν^D (Eq. (6)) and M_R^ν (Eq. (9)) and with an input value of $y \approx -1/17$ (Note that by flavor symmetry [12], we a priori expect $|y| \sim 1/10$) we get:

$$m(\nu_2) \approx (6-7) \times 10^{-3} \quad \text{eV (from seesaw)} \tag{a}$$
$$m(\nu_1) \approx (1 - few) \times 10^{-3}; \quad \text{thus } \Delta m_{12}^2 \approx (3-5) \times 10^{-5} \text{eV}^2 \tag{b}$$
$$\sin^2 2\theta_{\nu_e\nu_\mu}^{osc} \approx (0.5 - 0.7) \quad \text{(from non seesaw)} \tag{c}$$
$$\theta_{13} \lesssim (2-5) \times 10^{-2} \tag{d}$$

$$\tag{12}$$

While the results in Eq. (11) are compelling predictions of the model, the LMA-compatible solution for $\theta_{\nu_e\nu_\mu}^{osc}$ listed in (12)(c) should be regarded as a plausible and consistent possibility rather than as a compelling prediction of the framework.

The Majorana masses of the RH neutrinos ($N_{iR} \equiv N_i$) are given by [35]:

$$
\begin{aligned}
M_3 &\approx M_R \approx 4 \times 10^{14} \text{ GeV (1/2-2)}, \\
M_2 &\approx |y^2|M_3 \approx 10^{12} \text{ GeV(1/2-2)}, \\
M_1 &\approx |x - z^2|M_3 \sim (1/4\text{-}2)10^{-4}M_3 \\
&\sim 4 \times 10^{10} \text{ GeV(1/8} - 4).
\end{aligned}
\tag{13}
$$

where $y \approx -1/17$ and $x \sim z^2 \sim 10^{-4}(1/2 - 2)$ have been used, in accord with flavor-symmetry [12]. *Note that we necessarily have a hierarchical pattern for the light as well as the heavy neutrinos with normal hierarchy $m_1 \lesssim m_2 \ll m_3$ and $M_1 \ll M_2 \ll M_3$.*

Leaving aside therefore the question of the $\nu_e - \nu_\mu$ oscillation angle, it seems quite remarkable that all seven predictions in Eq.(11) agree with observations to within 10%. Particularly intriguing is the $(B-L)$-dependent group-theoretic correlation between V_{cb} and $\theta^{osc}_{\nu_\mu \nu_\tau}$, which explains simultaneously why one is small (V_{cb}) and the other is so large ($\theta^{osc}_{\nu_\mu \nu_\tau}$) [9, 12].

Why V_{cb} is small while $\theta^{osc}_{\nu_\mu \nu_\tau}$ is large?

A Comment is in order about this last feature. Often it has been remarked by several authors that while the "observed" near equality of m_b and m_τ at the GUT-scale supports quark-lepton unification, the sharp difference between V_{cb} versus $\theta^{osc}_{\nu_\mu \nu_\tau}$ disfavors such a unification. I believe that the truth is quite the opposite. This becomes apparent if one notices a simple group-theoretic property *of the minimal Higgs system* $\left(\mathbf{45_H}, \mathbf{16_H}, \mathbf{\overline{16}_H}, \mathbf{10_H}\right)$. While such a system makes SU(4)-color preserving family-symmetric contributions to fermion masses through $\langle \mathbf{10_H} \rangle$ (which yields $m_b^\circ = m_\tau^\circ$), *it can make SU(4)-color breaking $(B - L)$- dependent contribution denoted by "ϵ" (see Eq. (6)) only through the combination $\langle \mathbf{10_H} \rangle . \langle \mathbf{45_H} \rangle / M$, which, however, is family-antisymmetric.* As a result, the $(B - L)$-dependent contribution enters into the "23" and the "32" entries but not into the "33"-entry (see Eq. (6)).

With "ϵ" being hierarchical (of order 1/10), following diagonalization, this in turn means that the SU(4)-color breaking effect for the masses of the third family-fermions are small (of order ϵ^2) as desired to preserve the near equality $m_b^\circ \approx m_\tau^\circ$; but such breaking effects are necessarily large for the masses of the second family fermions (likewise for the first family), again just as desired to account for $m_\mu^\circ \neq m_s^\circ$. The SU(4)-color breaking effects are also large for the mixings between second and the third family fermions (arising from the "23" and "32" entries), which precisely explain why $V_{cb} \ll \theta^{osc}_{\nu_\mu \nu_\tau}$ and yet $m_b^\circ \approx m_\tau^\circ$.

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To be specific, it may be noted from the expressions for V_{cb} and $\theta^{osc}_{\nu_\mu \nu_\tau}$ in Eq. (11), that while the family asymmetric and $(B-L)$- dependent square root factors like $(\eta + \epsilon/\eta - \epsilon)^{1/2}$ suppress V_{cb}, if ϵ is relatively negative compared to η, the analogous factor $(\eta - 3\epsilon/\eta + 3\epsilon)^{1/2}$, necessarily enhances $\theta^{osc}_{\nu_\mu \nu_\tau}$ in a predictable manner for the same sign of ϵ relative to η (the magnitudes of η, σ and ϵ are of course fixed by quark-lepton masses [9]). In other words, this correlation between the suppression of V_{cb} and the enhancement of $\theta^{osc}_{\nu_\mu \nu_\tau}$ has come about due to the group theoretic property of $\langle \mathbf{10_H} \rangle . \langle \mathbf{45_H} \rangle / M$ which is proportional to $B - L$, but family-antisymmetric. Note this correlation would be absent if 126_H were used to introduce $(B-L)-$dependence because its contributions would be family-symmetric, and the corresponding square root factors would reduce to unity.

Another interesting point of the hierarchical BPW model is that with $|y|$ being hierarchical (of order $1/10$ as opposed to being of order 1) and $m(\nu_2)/m(\nu_3)$ being of order $1/5$–$1/10$, it as shown in Ref. [9] that *the mixing angle from the neutrino sector $\sqrt{m(\nu_2)/m(\nu_3)}$ necessarily add (rather than subtract) to the contribution from the charged lepton sector (see Eq. (11))*. As a result, in the BPW model, both charged lepton and neutrino-sectors give medium-large contribution (≈ 0.4) which add to naturally yield a maximal $\theta^{osc}_{\nu_\mu \nu_\tau}$. This thus becomes a simple and compelling prediction of the model, based essentially on the group theory of the minimal Higgs system in the context of SO(10) or G(224) and the hierarchical nature of the mass-matrices.[7].

The success of the model as regards the seven predictions listed above provides some confidence in the *gross pattern* of the Dirac mass matrices presented above and motivates the study of CP and flavor violations and baryogenesis within the same framework. This is what I do in the next sections.

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[7]The explanation of the largeness of $\theta^{osc}_{\nu_\mu \nu_\tau}$ together with the smallness of V_{cb} outlined above, based on medium-large contributions from the charged lepton and neutrino sectors, is quite distinct from alternative explanations. In paricular, in the lop-sided Albright-Barr model [28], the largeness of $\theta^{osc}_{\nu_\mu \nu_\tau}$ arises almost entirely from the lop-sidedness of the charged lepton mass matrix. This distinction between the BPW and the AB models leads to markedly different predictions for the rate of $\mu \rightarrow e\gamma$ decay in the two models (see remarks later).

5 CP and Flavor Violations in the SUSY SO(10)/G(224) Framework

5.1 Some Experimental Facts

On the experimental side there are now four well measured quantities reflecting CP and/or $\Delta F = 2$ flavor violations. They are:[8]

$$\Delta m_K, \ \epsilon_K, \ \Delta m_{B_d} \text{ and } S(B_d \to J/\Psi K_S) \tag{14}$$

where $S(B_d \to J/\Psi K_S)$ denotes the asymmetry parameter in $(B_d \text{ versus } \overline{B_d}) \to J/\Psi K_S$ decays. It is indeed remarkable that the observed values including the signs of all four quantities as well as the empirical lower limit on Δm_{B_s} can consistently be realized within the standard CKM model for a single choice of the Wolfenstein parameters [36]:

$$\bar\rho_W = 0.178 \ \pm \ 0.046; \ \bar\eta_W = 0.341 \ \pm \ 0.028 \ . \tag{15}$$

This fit is obtained using the observed values of $\epsilon_K = 2.27 \times 10^{-3}$, $V_{us} = 0.2240 \pm 0.0036$, $|V_{ub}| = (3.30 \pm 0.24) \times 10^{-3}$, $|V_{cb}| = (4.14 \pm 0.07) \times 10^{-2}$, $|\Delta m_{B_d}| = (3.3 \pm 0.06) \times 10^{-13}$ GeV and $\Delta m_{B_d}/\Delta m_{B_s} > 0.035$, and allowing for uncertainties in the hadronic matrix elements of up to 15%. One can then predict the asymmetry parameter $S(B_d \to J/\Psi K_S)$ in the SM to be $\approx 0.685 \pm 0.052$ [36, 37]. This agrees remarkably well with the observed value $S(B_d \to J/\Psi K_S)_{expt.} = 0.734 \pm 0.054$, representing an average of the BABAR and BELLE results [38]. This agreement of the SM prediction with the data in turn poses a challenge for physics beyond the SM, especially for supersymmetric grand unified (SUSY GUT) models, as these generically possess new sources of CP and flavor violations beyond those of the SM.

5.2 Origin of CKM CP Violation in SO(10)/G(224)

At the outset I need to say a few words about the origin of CP violation within the G(224)/SO(10)-framework presented above. Following Ref. [9], the discussion so far has ignored, for the sake of simplicity, possible CP violating phases in the parameters (σ, η, ϵ, η', ϵ', $\zeta_{22}^{u,d}$, y, z, and x) of the Dirac and Majorana mass matrices [Eqs. (6, and (9)]. In

---

[8]ϵ'_K reflecting direct $\Delta F = 1$ CP violation is well measured, but its theoretical implications are at present unclear due to uncertainties in the matrix element. We discuss this later.

general, however, these parameters can and generically will have phases [39]. Some combinations of these phases enter into the CKM matrix and define the Wolfenstein parameters ρ_W and η_W, which in turn induce CP violation by utilizing the standard model interactions.It should be stressed, however, *that the values of $(\bar{\rho}_W, \bar{\eta}_W)$ obtained this way from a given pattern of mass matrices based on SO(10) (as in Eq. (6)) need not agree (even nearly) with the SM-based phenomenological values shown in Eq. (15), for any choice of phases of the parameters of the mass-matrices.* That in turn would pose a challenge for the SO(10)-model in question as to whether it can adequately describe observed CP and flavor violations (see discussion below).

We choose to diagonalize the quark mass matrices M_u and M_d at the GUT scale $\sim 2 \times 10^{16}$ GeV, by bi-unitary transformations - i.e.

$$M_d^{diag} \; = \; X_L^{d\dagger} M_d X_R^d \text{ and } M_u^{diag} \; = \; X_L^{u\dagger} M_u X_R^u \tag{16}$$

with phases of $q_{L,R}^i$ chosen such that the eigenvalues are real and positive and that the CKM matrix V_{CKM} (defined below) has the Wolfenstein form [40]). Approximate analytic expressions for $X_{L,R}$ are given in Ref. [10].

The CKM elements in the Wolfenstein basis are given by the matrix $V_{CKM} = e^{-i\alpha}(X_L^{u\dagger} X_L^d)$, where $\alpha = (\phi_{\sigma-\epsilon} - \phi_{\eta-\epsilon}) - (\phi_{\epsilon'} - \phi_{\eta'+\epsilon'})$.

5.3 SUSY CP and Flavor Violations

SUSY Breaking

As is well known, since the model is supersymmetric, non-standard CP and flavor violations would generically arise in the model through sfermion/gaugino quantum loops involving scalar $(mass)^2$ transitions [41]. The latter can either preserve chirality (as in $\tilde{q}_{L,R}^i \to \tilde{q}_{L,R}^j$) or flip chirality (as in $\tilde{q}_{L,R}^i \to \tilde{q}_{R,L}^j$). Subject to our assumption on SUSY breaking (specified below), it would turn out that these scalar $(mass)^2$ parameters get completely determined within our model by the fermion mass-matrices, and the few parameters of SUSY breaking.

We assume that flavor-universal soft SUSY-breaking is transmitted to the SM-sector at a messenger scale M^*, where $M_{GUT} < M^* \leq M_{string}$. This may naturally be realized e.g. in models of mSUGRA [42], or gaugino-mediation [43]. With the assumption of extreme universality as in CMSSM, supersymmetry introduces five parameters at the scale M^*:

$$m_o, m_{1/2}, A_o, \tan\beta \text{ and } sgn(\mu).$$

For most purposes, we will adopt this restricted version of SUSY breaking with the added restriction that $A_o = 0$ at M* [43]. However, we will not insist on strict Higgs-squark-slepton mass universality. Even though we have flavor preservation at M*, flavor violating scalar (mass)^2–transitions and A-terms arise in the model through RG running from M* to M_{GUT} and from M_{GUT} to the EW scale. As described below, we thereby have *three sources* of flavor violation.

(i) RG Running of Scalar Masses from M* to M$_{\text{GUT}}$.

With family universality at the scale M*, all sfermions have the mass m$_o$ at this scale and the scalar (mass)^2 matrices are diagonal. Due to flavor dependent Yukawa couplings, with $h_t = h_b = h_\tau (= h_{33})$ being the largest, RG running from M* to M_{GUT} renders the third family lighter than the first two (see e.g. [44]) by the amount:

$$\Delta\hat{m}_{\tilde{b}_L}^2 = \Delta\hat{m}_{\tilde{b}_R}^2 = \Delta\hat{m}_{\tilde{\tau}_L}^2 = \Delta\hat{m}_{\tilde{\tau}_R}^2 \equiv \Delta \approx -\left(\frac{30m_o^2}{16\pi^2}\right)h_t^2\ ln(M^*/M_{GUT})\ . \qquad (17)$$

Note the large coefficient "30", which is a consequence of SO(10). The factor 30→12 for the case of G(224). The squark and slepton (mass)^2 matrices thus have the form $\tilde{M}^{(o)} = \text{diag}(m_o^2, m_o^2, m_o^2 - \Delta)$. Transforming $\tilde{M}^{(o)}$ by $X_{L,R}^f$, which diagonalize fermion mass-matrices, i.e. evaluating $X_L^{f\dagger}(\tilde{M}^{(o)})_{LL} X_L^f$ and similarly for L→R, where f = u, d, l, introduces off-diagonal elements in the so-called SUSY basis (at the GUT-scale) given by:

$$(\hat{\delta}_{LL,RR}^f)_{ij} = \left(X_{L,R}^{f\dagger}(\tilde{M}^{(o)})X_{L,R}^f\right)_{ij}/m_{\tilde{f}}^2 \qquad (18)$$

These induce flavor and CP violating transitions $\tilde{q}_{L,R}^i \rightarrow \tilde{q}_{L,R}^j$ and $\tilde{l}_{L,R}^i \rightarrow \tilde{l}_{L,R}^j$. Note that these transitions depend upon the matrices $X_{L,R}^f$, which are of course determined by the entries (including phases) in the fermion mass matrices (Eq. (6)). Here m$_{\tilde{f}}$ denotes an average squark or slepton mass (as appropriate) and the hat signifies GUT-scale values.

(ii) RG Running of the $A-$parameters from M* to M$_{\text{GUT}}$.

Even if $A_o = 0$ at the scale M* (as we assume for concreteness, see also [43]). RG running from M* to M$_{\text{GUT}}$ induces $A-$parameters at M$_{\text{GUT}}$, involving the SO(10)/G(224) gauginos and yukawa couplings [44]; these yield chirality flipping transitions $\tilde{q}_{L,R}^i \rightarrow \tilde{q}_{R,L}^j$ and $(\tilde{l}_{L,R}^i \rightarrow \tilde{l}_{R,L}^j)$. Because of large SO(10) Casimirs, these induced A-terms arising from post-GUT physics are large even if $\ln(M^*/M_{GUT}) \approx 1$. The chirality flipping transition

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angles are given by:

$$(\delta^f_{LR})_{ij} \equiv (A^f_{LR})_{ij} \left(\frac{v_f}{m^2_{\tilde{f}}}\right). \tag{19}$$

Here f = u, d, l. The matrices A^{ij}_{LR} are given explicitly in Refs. [10] and [11]. Note that these induced A-terms are also completely determined by the fermion mass matrices, for any given choice of the universal SUSY parameters (m_o, $m_{1/2}$, $\tan\beta$ and M^*).

(iii) Flavor Violation Through RG Running From M_{GUT} to m_W in MSSM : It is well known that, even with universal masses at the GUT scale, RG running from M_{GUT} to m_W in MSSM, involving contribution from the top Yukawa coupling, gives a significant correction to the mass of $\tilde{b}'_L = V_{td}\tilde{d}_L + V_{ts}\tilde{s}_L + V_{tb}\tilde{b}_L$, which is not shared by the mass-shifts of $\tilde{b}_R$, $\tilde{d}_{L,R}$ and $\tilde{s}_{L,R}$. This in turn induces flavor violation. Here, $\tilde{d}_L$, $\tilde{s}_L$ and $\tilde{b}_L$ are the SUSY partners of the physical d_L, s_L and b_L respectively. The differential mass shift of $\tilde{b}'_L$ arising as above, may be expressed by an effective Lagrangian [45]: $\Delta\mathcal{L} = -(\Delta m'^2_L)\tilde{b}'^*_L\tilde{b}'_L$, where

$$\Delta m'^2_L = -3/2m^2_o\eta_t + 2.3A_o m_{1/2}\eta_t(1-\eta_t) - (A^2_o/2)\eta_t(1-\eta_t) + m^2_{1/2}(3\eta^2_t - 7\eta_t) . \tag{20}$$

Here $\eta_t = (h_t/h_f) \approx (m_t/v\sin\beta)^2(1/1.21) \approx 0.836$ for $\tan\beta = 3$. Expressing $\tilde{b}'_L$ in terms of down-flavor squarks in the SUSY basis as above, Eq. (20) yields new contributions to off diagonal squark mixing. Normalizing to m^2_{sq}, they are given by

$$\delta^{'(12,13,23)}_{LL} = \left(\frac{\Delta m'^2_L}{m^2_{sq}}\right)(V^*_{td}V_{ts},\ V^*_{td}V_{tb},\ V^*_{ts}V_{tb}) . \tag{21}$$

The net chirality preserving squark $(mass)^2$ off-diagonal elements at m_W are then obtained by adding the respective GUT-scale contributions from Eqs. (18) to that from Eq. (21). They are:

$$\delta^{ij}_{LL} = \hat{\delta}^{ij}_{LL} + \delta'^{ij}_{LL}; \quad \delta^{ij}_{RR} = \hat{\delta}^{ij}_{RR} \tag{22}$$

5.4 The Challenge for SUSY SO(10)/G(224)

The interesting point is that the net values including phases of the off-diagonal squark-mixings, arising from the three sources listed above, and thereby the flavor and CP violations induced by them, are entirely determined within our approach by the entries in the quark mass-matrices and the choice of the universal SUSY parameters (m_0, $m_{1/2}$, M^*,

$\tan\beta$ and $\text{sgn}(\mu)$). Within the $G(224)/SO(10)$ framework presented in Sec. 4, the quark mass-matrices are however tightly constrained by our considerations of fermion masses and neutrino-oscillations.

The question thus arises: *Can observed CP and/or flavor-violations in the quark and lepton sectors (including the empirical limits on some of these) emerge consistently within the $G(224)/SO(10)$-framework, for* any *choice of phases in the fermion mass-matrices of Eq. (6), while preserving all its successes with respect to fermion masses and neutrino oscillations?*

This is indeed a *non-trivial challenge* to meet within the $SO(10)$ or $G(224)$-framework, since the constraints from both CP and flavor violations on the one hand and fermion masses and neutrino oscillations on the other hand are severe.

To be specific, the fact that all four entities (Δm_K, ϵ_K, Δm_{B_d} and $S(B_d \to J/\psi K_S)$) can be realized consistently in accord with experiments within the standard CKM model for a single choice of the Wolfenstein parameters $\bar{\rho}_W$ and $\bar{\eta}_W$ (Eq. (15)) strongly suggests *that even for the SUSY SO(10)/G(224)-model, the corresponding SM-contributions, at least to these four entities, should be the dominant ones, with SUSY contributions being sub-dominant or small.*[9] This in turn means that there should exist a choice of the parameters of the SO(10)-based mass matrices (like σ, η, ϵ, ϵ' etc.), viewed in general as complex, for which not only (a) the fermion masses and (b) the CKM mixings $|V_{ij}|$ should be described correctly (as in Eq.(11)), but also (c) the Wolfenstein parameters $\bar{\rho}'_W$ and $\bar{\eta}'_W$ derived from the SO(10)-based mass-matrices should be close to the phenomenological SM values (Eq. (15)). *A priori*, a given SO(10)-model, with a specified pattern for fermion mass matrices, may not in fact be able to satisfy all three constraints (a), (b) and (c) simultaneously.[10]

5.5   The Results

Without further elaboration, I will now briefly summarize the main results of Refs. [10] and [11].

(1) Allowing for phases ($\sim 1/10$ to $\sim 1/2$) in the parameters η, σ, ϵ' and ζ_{22}^d of the

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[9]The alternative of SUSY-contributions being relatively important compared to the SO(10)-based SM contributions and correcting for its pitfalls in just the right way for each of these four entities appear to be rather contrived and may require arbitrary adjustment of the many MSSM parameters. Such a scenario would at the very least mean that the good agreement between the SM-predictions and experiments is fortuitous.

[10]For a discussion of the difficulties in this regard within a recently proposed SO(10)-model see e.g. Ref. [46].

$G(224)/SO(10)$-framework (see Eq. (6)) we found that there do exist solutions which yield masses and mixings of quarks and leptons including neutrinos, all in good accord with observations (to within 10 %), and at the same time yield the following values for the Wolfenstein parameters (see Ref. [10] for details):

$$\bar{\rho}'_W \approx 0.15, \ \bar{\eta}'_W \approx 0.37. \quad (SO(10)/G(224) - \text{model}) \tag{23}$$

The prime here signifies that these are the values of $\bar{\rho}_W$ and $\bar{\eta}_W$ which are derived (for a suitable choice of phases in the parameters of the fermion mass matrices) from within the structure of the SO(10)-based mass-matrices (Eq. (6)). The corresponding phenomenological values are listed in Eq. (15). Note, as desired, the $G(224)/SO(10)$-framework presented here has turned out to be capable of yielding $\bar{\rho}'_W$ and $\bar{\eta}'_W$ close to the SM-values of $\bar{\rho}_W$ and $\bar{\eta}_W$ while preserving the successes with respect to fermion masses and neutrino oscillations as in Sec. 4. As mentioned above, this is indeed a non-trivial but most desirable feature.

(2) Including both the SM-contribution (with $\bar{\rho}'_W$ and $\bar{\eta}'_W$ as above) and the SUSY-contribution (with a plausible choice of the spectrum-e.g. $m_{sq} \approx (0.8 - 1)$ TeV and $x = (m_{\tilde{g}}^2/m_{sq}^2) \approx 0.6 - 0.8$), we obtain [10]:

$$(\Delta m_K)_{shortdist} \approx 3 \times 10^{-15} \text{ GeV};$$
$$\epsilon_K \approx (2 \text{ to } 2.5) \times 10^{-3};$$
$$\Delta m_{B_d} \approx (3.5 \text{ to } 3.6) \times 10^{-13} \text{ GeV};$$
$$S(B_d \to J/\Psi K_s) \approx 0.68 - 0.74. \tag{24}$$

We have used $\hat{B}_K = 0.86$ and $f_{Bd}\sqrt{\hat{B}_{Bd}} = 215$ MeV (see [36]). Now all four on which there is reliable data are in good agreement with observations (within 10%). The spectrum of $(m_{sq}, m_{\tilde{g}})$ considered above can be realized, for example for a choice of $(m_0, m_{1/2}) \approx (600, 220)$ GeV. For a more complete presentation of the results involving other choices of $(m_o, \ m_{1/2})$, and a discussion on the issue of consistency with WMAP results on the LSP as cold dark matter, see Refs. [10] and [11].

In all these cases, the SUSY-contribution turns out to be rather small ($\lesssim 5\%$ in amplitude), except however for ϵ_K, for which it is sizable ($\approx 20 - 30\%$) and has opposite sign, compared to the SM-contribution. Had the SUSY contribution to ϵ_K been positive relative to the SM-contribution, ϵ_K(total) would have been too large ($\approx (3.1 - 3.5) \times 10^{-3}$), in strong

disagreement with the observed value of 2.27×10^{-3}, despite the uncertainty in $\hat{B}_K$. In short, *the SUSY contribution of the model to ϵ_K has just the right sign and nearly the right magnitude to play the desired role.* This seems to be an intriguing feature of the model.

We thus see that the SUSY G(224) or SO(10)-framework (remarkably enough) has met all the challenges so far in being able to reproduce the observed features of both CP and quark-flavor violations as well as fermion masses and neutrino-oscillations!

Other Predictions

Other predictions of the model which incorporate contributions from δ_{LL}^{23}, δ_{RR}^{23}, δ_{LR}^{23} and δ_{RL}^{23}, include (see Ref. [10] for details):

$$S(B_d \to \phi K_S)(Tot \approx SM) \approx 0.65 - 0.73 \tag{25}$$

$$\Delta m_{B_s}(Tot \approx SM) \approx 17.3 \ ps^{-1} \Big(\frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{245 MeV}\Big)^2 \ . \tag{26}$$

$$A(b \to s\gamma)_{SUSY} \approx (1-5)\% \ \text{of} \ A(b \to s\gamma)_{SM} \tag{27}$$

$$Re(\epsilon'/\epsilon)_{SUSY} \approx +(8.8 \times 10^{-4})(B_G/4)(5/\tan\beta) \ . \tag{28}$$

Particularly interesting is the prediction of the model that the asymmetry parameter $S(B_d \to \phi K_S)$ should be close to the SM value of $\approx 0.70 \pm 0.10$. At present, there is conflicting data: $S(B_d \to \phi K_S) = (+0.50 \pm 0.25^{+0.07}_{-0.04})_{BaBar}; (+0.06 \pm 0.33 \pm 0.09)_{BELLE}$ [47][11]. It will thus be extremely interesting to see both from the point of view of the present model and the SM whether the true value of $S(B_d \to \phi K_S)$ will turn out to be close to the SM prediction or not.

EDM's

For a representative choice of $(m_o, \ m_{1/2}) = (600, 300)$ GeV (i.e. $m_{sq} = 1$ TeV, $m_{\tilde{g}} = 900$ GeV, $m_{\tilde{l}} = 636$ GeV and $m_{\tilde{B}} = 120$ GeV), the induced A-terms (see Eq. (19)) lead to [10]:

$$(d_n)_{A_{ind}} = (1.6, 1.08) \times 10^{-26} ecm \ \text{for} \ \tan\beta = (5, 10) \ . \tag{29}$$

$$(d_e)_{A_{ind}} = \frac{1.1 \times 10^{-28}}{\tan\beta} \ ecm \ . \tag{30}$$

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[11]At the time of completing this manuscript, the BELLE group reported a new value of $S(B_d \to \phi K_S) = +0.44 \pm 0.27^{\pm}0.05$ at the 2005 Lepton-Photon Symposium [48]. This value is close to that reported by BaBar and enhances the possibilty of the true value being close to the SM value.

Given the experimental limits $d_n < 6.3 \times 10^{-26}$ e cm [49] and $d_e < 4.3 \times 10^{-27}$ e cm [50], we see that the predictions of the model (arising only from the induced A-term contributions) especially for the EDM of the neutron is in an extremely interesting range suggesting that it should be discovered with an improvement of the current limit by a factor of about 10.

6 Lepton Flavor Violation in SUSY SO(10)/G(224)

It has been recognized for sometime that lepton flavor violating processes (such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu N \to eN$ etc.), can provide sensitive probes into new physics beyond the SM, especially that arising in SUSY grand-unification [41, 44, 51], and that too with heavy right-handed neutrinos [51]. In our case these get contributions from three sources:

(i) The slepton (mass)^2 elements $(\delta m^2)_{LL}^{ij}$ arising from RG-running of scalar masses from $M^* \to M_{GUT}$ in the context of SO(10)/G(224) (see Eq. (18)),

(ii) The chirality flipping slepton (mass)^2 elements $(\delta m^2)_{LR}^{ij}$ arising from A-terms induced through RG-running from $M^* \to M_{GUT}$ in the context of SO(10) or G(224) (see Sec. 5.3), and

(iii) $(\delta m'^2)_{LL}^{ij}$ arising from RG-running from M_{GUT} to the RH neutrino mass-scales M_{R_i} involving ν_R^i Dirac Yukawa couplings corresponding to Eq.(6) which (in the leading log approximation) yield:

$$(\delta_{LL}^l)_{ij}^{RHN} = \frac{-(3m_o^2 + A_o^2)}{8\pi^2} \sum_{k=1}^{3} (Y_N)_{ik}(Y_N^*)_{jk} \; ln(\frac{M_{GUT}}{M_{R_k}}) \; . \tag{31}$$

Note that the masses M_{R_i} of RH neutrinos are fairly well determined within the model (see Eq. (13)).

There is a vast literature on the subject of lepton flavor violation (LFV). (For earlier works see Ref. [44, 51]; and for a partial list of references including recent works see Ref. [52]). Most of the works in the literature have focused only on the contribution from the third source, involving the Yukawa couplings of the RH neutrinos, which is proportional to $\tan\beta$ in the amplitude. It turns out, however, that the contributions from the first two sources arising from post-GUT physics (i.e. $SO(10)$-running from M^* to M_{GUT}) are in fact the dominant ones for $\tan\beta \lesssim 10$, as long as $\ln(M^*/M_{GUT}) \gtrsim 1$. We consider the contribution from all three sources by summing the corresponding amplitudes, and by varying (m_0, $m_{1/2}$, $\tan\beta$

$(m_o,\ m_{1/2})//\tan\beta$	$\mathrm{Br}(\mu \to e\gamma)$		$\mathrm{Br}(\tau \to \mu\gamma)$	
	$\mu > 0$	$\mu < 0$	$\mu > 0$	$\mu < 0$
I (600, 300)//10	3.3×10^{-12}	9.8×10^{-12}	2.4×10^{-9}	3.1×10^{-9}
II (800, 250)//10	2.9×10^{-13}	1.7×10^{-12}	1.9×10^{-9}	1.9×10^{-9}
III (450, 300)//10	2.7×10^{-11}	4.6×10^{-11}	2.7×10^{-9}	5.6×10^{-9}
IV (500, 250)//10	5.9×10^{-12}	1.9×10^{-11}	4.8×10^{-9}	6.4×10^{-9}
V (100, 440)//10	1.02×10^{-8}	1.02×10^{-8}	8.3×10^{-8}	8.4×10^{-8}
VI (1000, 250)//10	1.6×10^{-13}	5.6×10^{-12}	9.5×10^{-10}	9.0×10^{-10}
VII (400, 300)//20	9.5×10^{-12}	3.8×10^{-11}	1.4×10^{-8}	1.8×10^{-8}

Table 1: Branching ratios of $l_i \to l_j\gamma$ for the SO(10) framework with $\kappa \equiv \ln(M^*/M_{GUT}) = 1$; $(m_o,\ m_{1/2})$ are given in GeV, which determine μ through radiative electroweak symmetry breaking conditions. The entries for $\mathrm{Br}(\mu \to e\gamma)$ for the case of G(224) would be reduced by a factor $\approx 4-6$ compared to that of SO(10) (see text).

and $sgn(\mu)$)

Here I present the predictions of the model for five different choices of $(m_o,\ m_{1/2})$ with $\tan\beta = 10$ or 20 and $\ln(M^*/M_{GUT}) = 1$, to indicate the nature of the predictions. (Results for a wider choice of parameters and a more detailed discussion may be found in Ref. [11]). We have set $(M_{R_1},\ M_{R_2},\ M_{R_3}) = (10^{10},\ 10^{12},\ 5\times10^{14})$ GeV (see Eq. (13)) and $A_o(M^*) = 0$. The predicted rates for G(224) are smaller than those for SO(10) approximately by a factor of 4 to 6 (see comments in Sec. 5). The results for SO(10) are presented in Table 1.

The following points regarding these results are worth noting:

(1) We find that the contribution due to the presence of the RH neutrinos[12] is about an order of magnitude smaller, *in the amplitude*, than those of the others arising from post-GUT physics (proportional to $\hat{\delta}^{ij}_{LL}$, δ^{ij}_{LR} and δ^{ij}_{RL}). The latter arise from RG running of the scalar

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[12]In the context of contributions due to the RH neutrinos alone, there exists an important distinction (partially observed by Barr, see Ref. [53]) between the hierarchical BPW form [9] and the lop-sided Albright-Barr (AB) form [28] of the mass-matrices. The amplitude for $\mu \to e\gamma$ from this source turns out to be proportional to the difference between the (23)-elements of the Dirac mass-matrices of the charged leptons and the neutrinos, with (33)-element being 1. This difference is (see Eq. (6)) is $\eta - \sigma \approx 0.041$, *which is naturally small for the hierarchical BPW model* (incidentally it is also V_{cb}), while it is order one for the lop-sided AB model. This means that the rate for $\mu \to e\gamma$ due to RH neutrinos would be about 600 times larger in the AB model than the BPW model (for the same input SUSY parameters). For a comparative study of the BPW and the AB models using processes such as $\mu \to e\gamma$ and edm's, see forthcoming paper by P. Rastogi [54].

masses and the A−parameters in the context of SO(10) or G(224) from M^* to M_{GUT}. It seems to us that the latter, which have commonly been omitted in the literature, should exist in any SUSY GUT model for which the messenger scale for SUSY-breaking is high ($M^* > M_{GUT}$), as in a mSUGRA model. *The inclusion of these new contributions to LFV processes arising from post-GUT physics, that too in the context of a predictive and realistic framework, is the distinguishing feature of the study carried out in Ref. [11].*[13]

(2) Owing to the general prominence of the new contributions from post-GUT physics, we see from table 1 that case V, (with low m_o and high $m_{1/2}$) is clearly excluded by the empirical limit on $\mu \to e\gamma$-rate (see Sec. 1). Case III is also excluded, for the case of SO(10), yielding a rate that exceeds the limit by a factor of about 2 (for $\kappa = \ln(M^*/M_{GUT}) \gtrsim 1$), though we note that for the case of G(224), Case III is still perfectly compatible with the observed limit (see remark below table 1). All the other cases (I, II, IV, VI, and VII), with medium or moderately heavy ($\gtrsim 500$ GeV) sleptons , are compatible with the empirical limit, even for the case of SO(10). The interesting point about these predictions of our model, however, is that $\mu \to e\gamma$ should be discovered, even with moderately heavy sleptons ($\sim 800 - 1000$ GeV), both for SO(10) and G(224), with improvement in the current limit by a factor of 10–100. Such an improvement is being planned at the forthcoming MEG experiment at PSI.

(3) We see from table 1 that $\tau \to \mu\gamma$ (leaving aside case V, which is excluded by the limit on $\mu \to e\gamma$), is expected to have a branching ratio in the range of 2×10^{-8} (Case VII) to about (1 or 2) $\times 10^{-9}$ (Case VI or II). The former may be probed at BABAR and BELLE, while the latter can be reached at the LHC or a super B factory. The process $\tau \to e\gamma$ would, however, be inaccessible in the foreseeable future (in the context of our model).

(4) The WMAP-Constraint: Of the cases exhibited in table 1, Case V ($m_o = 100$ GeV, $m_{1/2} = 440$ GeV) would be compatible with the WMAP-constraint on relic dark matter density, in the context of CMSSM, assuming that the lightest neutralino is the LSP and represents cold dark matter (CDM), accompanying co-annihilation mechanism. (See e.g. [55]). As mentioned above (see table 1), a spectrum like Case V, with low m_o and higher $m_{1/2}$, is however excluded in our model by the empirical limit on $\mu \to e\gamma$. *Thus we infer that*

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[13]For the sake of comparison, should one drop the post-GUT contribution by setting $M^* = M_{GUT}$, however, the predicted $Br(\mu \to e\gamma)$, based on RHN contributions only, would be reduced significantly in our model to e.g. (4.2, 2.9, and 8.6)$\times 10^{-15}$ for cases I, II and IV respectively.

in the context of our model CDM cannot be associated with the co-annihilation mechanism.

Several authors (see e.g. Refs. [56] and [57]), have, however considered the possibility that Higgs-squark-slepton mass universality need not hold even if family universality does. In the context of such non-universal Higgs mass (NUHM) models, the authors of Ref. [57] show that agreement with the WMAP data can be obtained over a wide range of mSUGRA parameters. In particular, such agreement is obtained for (m_ϕ/m_o) of order unity (with either sign) for almost all the cases (I, II, III, IV, VI and VII)[14], with the LSP (neutralino) representing CDM.[15] (Here $m_\phi \equiv sign(m_{H_{u,d}}^2)\sqrt{|m_{H_{u,d}}^2|}$, see [57]). All these cases (including Case III for G(224)) are of course compatible with the limit on $\mu \to e\gamma$.

(5) Coherent $\mu - e$ conversion in nuclei: In our framework, $\mu - e$ conversion (i.e. $\mu^- + N \to e^- + N$) will occur when the photon emitted in the virtual decay $\mu \to e\gamma^*$ is absorbed by the nucleus (see e.g. [58]). In such situations, there is a rather simple relation connecting the $\mu - e$ conversion rate with $B(\mu \to e\gamma)$: $B(\mu \to e\gamma)/(\omega_{conversion}/\omega_{capture}) = R \simeq (230 - 400)$, depending on the nucleus. For example, R has been calculated to be $R \simeq 389$ for ^{27}Al, 238 for ^{48}Ti and 342 for ^{208}Pb in this type of models. (These numbers were computed in [58] for the specific model of [44], but they should approximately hold for our model as well.) With the branching ratios listed in Table 1 ($\sim 10^{-11}$ to 10^{-13}) for our model, $\omega_{conversion}/\omega_{capture} \simeq (40\text{--}1) \times 10^{-15}$. The MECO experiment at Brookhaven is expected to have a sensitivity of 10^{-16} for this process, and thus will test our model.

In summary, lepton flavor violation is studied in [11] within a predictive SO(10)/G(224)-framework, possessing supersymmetry, that was proposed in Refs. [9, 10]. The framework seems most realistic in that it successfully describes five phenomena: (i) fermion masses and mixings, (ii) neutrino oscillations, (iii) CP violation, (iv) quark flavor-violations, as well as (v) baryogenesis via leptogenesis (see below) [5]. LFV emerges as an important prediction of this framework bringing no new parameters, barring the few flavor-preserving SUSY parameters.

Our results show that – (i) The decay $\mu \to e\gamma$ should be seen with improvement in the current limit by a factor of 10 – 100, even if sleptons are moderately heavy (~ 800 GeV, say); (ii) for the same reason, $\mu - e$ conversion ($\mu N \to eN$) should show in the planned MECO experiment, and (iii) $\tau \to \mu\gamma$ may be accessible at the LHC and a super B-factory.

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[14]We thank A. Mustafayev and H. Baer for private communications in this regard.

[15]We mention in passing that there may also be other posibilities for the CDM if we allow for either non-universal gaugino masses, or axino or gravitino as the LSP, or R-parity violation (with e.g. axion as the CDM).

7 Baryogenesis Via Leptogenesis Within the $G(224)/SO(10)$-Framework

The observed matter-antimatter asymmetry provides an important clue to physics at truly short distances. Given the existence of the RH neutrinos, as required by the symmetry $SU(4)$-color or $SU(2)_R$, possessing superheavy Majorana masses which violate B-L by two units, baryogenesis via leptogenesis [4,59] has emerged as perhaps the most viable and natural mechanism for generating the baryon asymmetry of the universe. The most interesting aspect of this mechanism is that it directly relates our understanding of the light neutrino masses to our own origin. The question of whether this mechanism can quantitatively explain the magnitude of the observed baryon-asymmetry depends however crucially on the Dirac as well as the Majorana mass-matrices of the neutrinos, including the phases and the eigenvalues of the latter-i.e. M_1, M_2 and M_3 (see Eq. (13)).

This question has been considered in a recent work [5] in the context of a realistic and predictive framework for fermion masses and neutrino oscillations, based on the symmetry $G(224)$ or $SO(10)$, as discussed in Sec. 4, with CP violation treated as in Sec. 5. It has also been discussed in a recent review [35]. Here I will primarily quote the results and refer the reader to Ref. [5] for more details especially including the discussion on inflation and relevant references.

The basic picture is this. Following inflation, the lightest RH neutrinos (N_1's) with a mass $\approx 10^{10}$ GeV (1/3 $-$ 3) are produced either from the thermal bath following reheating ($T_{RH} \approx$ few$\times 10^9$ GeV), or non-thermally directly from the decay of the inflaton [16] (with T_{RH} in this case being about $10^7 - 10^8$ GeV). In either case, the RH neutrinos having Majorana masses decay by utilizing their Dirac Yukawa couplings into both $l + H$ and $\bar{l} + \bar{H}$ (and corresponding SUSY modes), thus violating B-L. In the presence of CP violating phases, these decays produce a net lepton-asymmetry $Y_L = (n_L - n_{\bar{L}})/s$ which is converted to a baryon-asymmetry $Y_B = (n_B - n_{\bar{B}})/s = CY_L$ ($C \approx -1/3$ for MSSM) by the EW sphaleron effects. Using the Dirac and the Majorana mass-matrices of Sec. 4, with the introduction of CP-violating phases in them as discussed in Sec. 5, the lepton-asymmetry produced per N_1

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[16]In this case the inflaton can naturally be composed of the Higgs-like objects having the quantum numbers of the RH sneutrinos ($\tilde{\nu}_{RH}$ and $\tilde{\bar{\nu}}_{RH}$) lying in $(1, 2, 4)_H$ and $(1, 2, \bar{4})_H$ for $G(224)$ (or 16_H and $\bar{16}_H$ for $SO(10)$), whose VEV's break B-L and give Majorana masses to the RH neutrinos via the coupling shown in Eq. (8).

(or $(\tilde{N}_1 + \bar{\tilde{N}}_1)$-pair) decay is found to be [5]:

$$\epsilon_1 \approx \frac{1}{8\pi}\left(\frac{\mathcal{M}_u^0}{v}\right)^2 |(\sigma + 3\epsilon) - y|^2 \sin(2\phi_{21}) \times (-3)\left(\frac{M_1}{M_2}\right)$$

$$\approx -\left(2.0 \times 10^{-6}\right)\sin(2\phi_{21}) \times \left[\frac{(M_1/M_2)}{5 \times 10^{-3}}\right] \tag{32}$$

Here ϕ_{21} denotes an effective phase depending upon phases in the Dirac as well as Majorana mass-matrices (see Ref. [5]). Note that the parameters σ, ϵ, y and $(\mathcal{M}_u^0/v)$ are already determined within our framework (to within 10 %) from considerations of fermion masses and neutrino oscillations (see Sec. 4 and 5). Furthermore, from Eq. (13) we see that $M_1 \approx (1/3 - 3) \times 10^{10}$ GeV, and $M_2 \sim 2 \times 10^{12}$ GeV, thus $M_1/M_2 \approx (5 \times 10^{-3})(1/3 - 3)$. In short, leaving aside the phase factor, the RHS of Eq. (32) is pretty well determined within our framework (to within about a factor of 5), as opposed to being uncertain by orders of magnitude either way. *This is the advantage of our obtaining the lepton-asymmetry in conjunction with a predictive framework for fermion masses and neutrino oscillations.* Now the phase angle ϕ_{21} is uncertain because we do not have any constraint yet on the phases in the Majorana sector (M_R^ν). At the same time, since the phases in the Dirac sector are relatively large (see Sec. 5 and Ref. [10]), barring unnatural cancellation between the Dirac and Majorana phases, we would naturally expect $\sin(2\phi_{21})$ to be sizable-i.e. of order 1/10 to 1 (say).

The lepton-asymmetry is given by $Y_L = \kappa(\epsilon_1/g^*)$, where κ denotes an efficiency factor representing wash-out effects and g^* denotes the light degrees of freedom ($g^* \approx 228$ for MSSM). For our model, using recent discussions on κ from Ref. [60], we obtain: $\kappa \approx (1/18 - 1/60)$, for the thermal case, depending upon the "31" entries in the neutrino-Dirac and Majorana mass-matrices (see Ref. [5]). Thus, for the thermal case, we obtain:

$$(Y_B)_{thermal}/\sin(2\phi_{21}) \approx (10 - 30) \times 10^{-11} \tag{33}$$

where, for concreteness, we have chosen $M_1 \approx 4 \times 10^9$ GeV and $M_2 \approx 1 \times 10^{12}$ GeV, in accord with Eq. (13). In this case, the reheat temperature would have to be about few $\times 10^9$ GeV so that N_1's can be produced thermally. We see that the derived values of Y_B can in fact account for the recently observed value $(Y_B)_{WMAP} \approx (8.7 \pm 0.4) \times 10^{-11}$ [61], for a natural value of the phase angle $\sin(2\phi_{21}) \approx (1/3 - 1)$. As discussed below, the case of non-thermal leptogenesis can allow even lower values of the phase angle. It also typically

λ	10^{-5}	10^{-6}
m_{infl} GeV	3×10^{11}	3×10^{10}
T_{RH} GeV	$(5.3 - 1.8) \times 10^7$	$(17 - 5.6) \times 10^6$
$\frac{Y_B \times 10^{11}}{\sin(2\phi_{21})}$	$(100 - 10)$	$(300 - 33)$

Table 2: Baryon Asymmetry For Non-Thermal Leptogenesis

yields a significantly lower reheat temperature ($\sim 10^7 - 10^8$ GeV) which may be in better accord with the gravitino-constraint.

For the non-thermal case, to be specific one may assume an effective superpotential [62]: $W_{eff}^{infl} = \lambda S(\bar{\Phi}\Phi - M^2)+$ (non-ren. terms) so as to implement hybrid inflation; here S is a singlet field and Φ and $\bar{\Phi}$ are Higgs fields transforming as $(1,2,4)$ and $(1,2,\bar{4})$ of $G(224)$ which break B-L at the GUT scale and give Majorana masses to the RH neutrinos. Following the discussion in [62], [5], one obtains: $m_{infl} = \sqrt{2}\lambda M$, where $M =<$ $(1,2,4)_H >\approx 2 \times 10^{16}$ GeV; $T_{RH} \approx (1/7)(\Gamma_{infl}M_{Pl})^{1/2} \approx (1/7)(M_1/M)(m_{infl}M_{Pl}/8\pi)^{1/2}$ and $Y_B \approx -(1/2)(T_{RH}/m_{infl})\varepsilon_1$. Taking the coupling λ in a plausible range $(10^{-5} - 10^{-6})$ (which lead to the desired reheat temperature, see below) and the asymmetry-parameter ε_1 for the $G(224)/SO(10)$-framework as given in Eq. (32), the baryon-asymmetry Y_B can then be derived. The values for Y_B thus obtained are listed in Table 2.

The variation in the entries correspond to taking $M_1 = (2 \times 10^{10}$ GeV$)(1 - 1/3)$ with $M_2 = (2 \times 10^{12})$ GeV in accord with Eq. (13). We see that for this case of non-thermal leptogenesis, one quite plausibly obtains

$$(Y_B)_{Non-Thermal} \approx (8 - 9) \times 10^{-11} \tag{34}$$

in full accord with the WMAP data, for natural values of the phase angle $\sin(2\phi_{21}) \approx (1/3 - 1/10)$, and with T_{RH} being as low as 10^7 GeV $(2 - 1/2)$. Such low values of the reheat temperature are fully consistent with the gravitino-constraint for $m_{3/2} \approx 400$ GeV -1 TeV (say), even if one allows for possible hadronic decays of the gravitinos for example via $\gamma\tilde{\gamma}$-modes [63].

In summary, I have presented two alternative scenarios (thermal as well as non-thermal) for inflation and leptogenesis. We see that the $G(224)/SO(10)$-framework provides *a simple and unified description* of not only fermion masses, neutrino oscillations (consistent with maximal atmospheric and large solar oscillation angles) *and* CP violation, but also of baryo-

genesis via leptogenesis, in either scenario. Each of the following features - (a) the existence of the RH neutrinos, (b) B-L local symmetry, (c) $SU(4)$-color, (d) the SUSY unification scale, (e) the seesaw mechanism, and (f) the pattern of $G(224)/SO(10)$ mass-matrices allowed in the minimal Higgs system (see Sec. 4)-have played crucial roles in realizing this *unified and successful description.*

8 Proton Decay

Perhaps the most dramatic prediction of grand unification is proton decay. I have discussed proton decay in the context of the SUSY $SO(10)/G(224)$-framework presented here in some detail in recent reviews [12, 35] which are updates of the results obtained in [9]. Here, I will present only the salient features and the updated results. In SUSY unification there are in general three distinct mechanisms for proton decay.

1. **The familiar d=6 operators** mediated by X and Y gauge bosons of $SU(5)$ and $SO(10)$ As is well known, these lead to $e^+\pi^0$ as the dominant mode with a lifetime $\approx 10^{35.3\pm1}$ yrs.

2. **The "standard" $d = 5$ operators** [64] which arise through the exchange of the color-triplet Higgsinos which are in the $5_H + \bar{5}_H$ of $SU(5)$ or 10_H of $SO(10)$. These operators require (for consistency with proton lifetime limits) that the color-triplets be made superheavy while the EW-doublets are kept light by a suitable doublet-triplet splitting mechanism (for $SO(10)$, see Ref. [25]. They lead to dominant $\bar{\nu}K^+$ and comparable $\bar{\nu}\pi^+$ modes with lifetimes varying from about 10^{29} to 10^{34} years, depending upon a few factors, which include the nature of the SUSY-spectrum and the matrix elements (see below). In the present context, see [9, 12, 65]. Some of the original references on contributions of standard $d = 5$ operators to proton decay may be found in [66–72]

3. **The so called "new" $d = 5$ operators** [9,73] which can generically arise through the exchange of color-triplet Higgsinos in the Higgs multiplets like $(16_H + \bar{16}_H)$ of $SO(10)$. Such exchanges are possible by utilizing the joint effects of (a) the couplings given in Eq. (8) which assign superheavy Majorana masses to the RH neutrinos through the VEV of $\bar{16}_H$, and (b) the coupling of the form $g_{ij}16_i16_j16_H16_H/M$ (see Eq. (7))

which are needed, at least for the minimal Higgs-system, to generate CKM-mixings. These operators also lead to $\bar{\nu}K^+$ and $\bar{\nu}\pi^+$ as the dominant modes, and they can quite plausibly lead to lifetimes in the range of $10^{32} - 10^{34}$ yrs [see below]. These operators, though most natural in a theory with Majorana masses for the RH neutrinos, have been invariably omitted in the literature.

One distinguishing feature of the new $d = 5$ operator is that they directly link proton decay to neutrino masses via the Majorana masses of the RH neutrinos. *The other, and perhaps most important, is that these new $d = 5$ operators can induce proton decay even when the $d = 6$ and standard $d = 5$ operators mentioned above are absent.* This is what could happen if the string theory [17] or a higher dimensional GUT-theory [74] leads to an effective $G(224)$-symmetry in $4D$, which would be devoid of both X and Y gauge bosons and the dangerous color-triplets in the 10_H of $SO(10)$. *By the same token, for an effective $G(224)$-theory, these new $d = 5$ operators can become the sole and viable source of proton decay leading to lifetimes in an interesting range (see below).*

Our study of proton decay carried out in Ref. [9] and updated in [65] and [12] has a few distinctive features: (i) It is based on a *realistic framework for fermion masses and neutrino oscillations*, as discussed in Sec. 4;(ii) It includes the *new $d = 5$ operators* in addition to the standard $d = 5$ and $d = 6$ operators; (iii) It restricts *GUT-scale threshold-corrections* to $\alpha_3(m_Z)$ so as to be in accord with the demand of "natural" coupling unification and thereby restricts M_{eff} that controls the strength of the standard $d = 5$ operators; and (iv) It allows for the ESSM extension [75] of MSSM motivated on several grounds (see e.g. [75] and [65]), which introduces two vectorlike families in $16 + \overline{16}$ of $SO(10)$ with masses of order 1 TeV, in addition to the three chiral families.

Guided by recent calculation based on quenched lattice QCD in the continuum limit [76] and renormalization factors A_L and A_s for d = 5 as in [77], we take (see Ref. [12] for details): $|\beta_H| \approx |\alpha_H| \approx (0.009 \text{ GeV}^3)(1/\sqrt{2} - \sqrt{2})$; $m_{\tilde{q}} \approx m_{\tilde{l}} \approx 1.2$ TeV $(1/2 - 2)$; $(m_{\tilde{W}}/m_{\tilde{q}}) = 1/6(1/2 - 2)$; $M_{H_C}(\min SU(5)) \leq 10^{16}$ GeV, $A_L \approx 0.32$, $A_S \approx 0.93$, $\tan \beta \leq 3$; $M_X \approx M_Y \approx 10^{16}$ GeV $(1 \pm 25\%)$, and $A_R(d = 6, e^+ \pi^0) \approx 3.4$.

The theoretical predictions for proton decay for the cases of minimal SUSY $SU(5)$, SUSY $SO(10)$ and $G(224)$-models developed in Secs. 3 and 4, are summarized in Table 3. They are

$$\left.\frac{\text{SUSY } SU(5)}{\text{MSSM (std. } d=5)}\right\} \quad \Gamma^{-1}(p \to \bar{\nu} K^+) \quad \leq \quad 1.2 \times 10^{31} \text{ yrs} \quad \begin{pmatrix} \text{Excluded by} \\ \text{SuperK} \end{pmatrix} \qquad (35)$$

$$\left.\frac{\text{SUSY } SO(10)}{\text{MSSM (std. } d=5)}\right\} \quad \Gamma^{-1}(p \to \bar{\nu} K^+) \quad \leq \quad 1 \times 10^{33} \text{ yrs} \quad \begin{pmatrix} \text{Tightly constrained} \\ \text{by SuperK} \end{pmatrix} (36)$$

$$\left.\frac{\text{SUSY } SO(10)}{\text{ESSM (std. } d=5)}\right\} \quad \begin{matrix} \Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{Med.}} \\ \Gamma^{-1}(p \to \bar{\nu} K^+) \end{matrix} \quad \begin{matrix} \approx \\ \lesssim \end{matrix} \quad \begin{matrix} (1\text{--}10) \times 10^{33} \text{ yrs} \\ 10^{35} \text{ yrs} \end{matrix} \quad \begin{pmatrix} \text{Fully SuperK} \\ \text{Compatible} \end{pmatrix} (37)$$

$$\left.\frac{\text{SUSY } G(224)/SO(10)}{\text{MSSM or ESSM (new } d=5)}\right\} \quad \begin{matrix} \Gamma^{-1}(p \to \bar{\nu} K^+) \\ B(p \to \mu^+ K^0) \end{matrix} \quad \begin{matrix} \lesssim \\ \approx \end{matrix} \quad \begin{matrix} 2 \times 10^{34} \text{ yrs} \\ (1-50)\% \end{matrix} \quad \begin{pmatrix} \text{Fully Compatible} \\ \text{with SuperK} \end{pmatrix} (38)$$

$$\left.\frac{\text{SUSY } SU(5) \text{ or } SO(10)}{\text{MSSM } (d=6)}\right\} \quad \Gamma^{-1}(p \to e^+ \pi^0) \quad \approx \quad 10^{35\pm1} \text{ yrs} \quad \begin{pmatrix} \text{Fully Compatible} \\ \text{with SuperK} \end{pmatrix} (39)$$

Table 3: A Summary of Results on Proton Decay

obtained by following the procedure as in [9, 12, 65] and using the parameters as mentioned above.[17]

It should be stressed that the upper limits on proton lifetimes given in Table 3 are quite conservative in that they are obtained (especially for the top two cases) by stretching the uncertainties in the matrix element and the SUSY spectra to their extremes so as to prolong proton lifetimes. In reality, the lifetimes should be shorter than the upper limits quoted above.

Now the experimental limits set by SuperK studies are as follows [78]:

$$\Gamma^{-1}(p \to e^+ \pi^0)_{\text{expt}} \geq 6 \times 10^{33} \text{ yrs}$$
$$\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{expt}} \geq 1.9 \times 10^{33} \text{ yrs} \qquad (40)$$

The following comments are in order.

1. By comparing the upper limit given in Eq. (35) with the experimental lower limit, we see that the *minimal* SUSY $SU(5)$ with the conventional MSSM spectrum is clearly excluded by a large margin by proton decay searches. This is in full agreement with the conclusion reached by other authors (see e.g. Ref. [72]).[18]

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[17]The chiral Lagrangian parameter $(D+F)$ and the renormalization factor A_R entering into the amplitude for $p \to e^+ \pi^0$ decay are taken to be 1.25 and 3.4 respectively.

[18]See, however, Refs. [79] and [80], where attempts are made to save minimal SUSY SU(5) by a set of scenarios. These

2. By comparing Eq. (36) with the empirical lower limit, we see that the case of MSSM embedded in $SO(10)$ is already tightly constrained to the point of being disfavored by the limit on proton lifetime. The constraint is of course augmented by our requirement of *natural coupling unification*, which prohibits accidental large cancelation between different threshold corrections (see [9]).

3. In contrast to the case of MSSM, that of ESSM [75] embedded in $SO(10)$, which has been motivated on several grounds[19], is fully compatible with the SuperK limit (see Eq. (37)). In this case, $\Gamma_{\text{Med}}^{-1}(p \to \bar{\nu}K^+) \approx 10^{33} - 10^{34}$ yrs, given in Eq. (37), corresponds to the parameters involving the SUSY spectrum and the matrix element β_H being in the *median range*, close to their central values.

4. We see from Eq. (38) that the contribution of the new operators [73] related to the Majorana masses of the RH neutrinos (which is the same for MSSM and ESSM and is independent of $\tan\beta$) is fully compatible with the SuperK limit. These operators can quite naturally lead to proton lifetimes in the range of $10^{33} - 10^{34}$ yrs with an upper limit of about 2×10^{34} yrs.

In summary for this section, within the $SO(10)/G(224)$ framework and with the inclusion of the standard as well as the new $d = 5$ operators, one obtains (see Eqs. (36)–(40)) a conservative upper limit on proton lifetime given by:

$$\tau_{\text{proton}} \lesssim (1/3 - 2) \times 10^{34} \text{ yrs} \quad \begin{pmatrix} \text{SUSY} \\ SO(10)/G(224) \end{pmatrix} \tag{41}$$

with $\bar{\nu}K^+$ and $\bar{\nu}\pi^+$ being the dominant modes and quite possibly μ^+K^0 being prominent.

The $e^+\pi^0$-mode induced by gauge boson-exchanges should have an inverse decay rate in the range of $10^{34} - 10^{36}$ years (see Eq. (35)). The implication of these predictions for a next-generation detector is noted in the next section.

include a judicious choice of sfermion mixings, higher dimensional operators and squarks of first two families having masses of order 10 TeV.

[19]The case of ESSM, which introduces two vector like families, i.e. $16 + \overline{16}$ of SO(10), with a mass of order 1 TeV, has been motivated by a number of considerations independently of proton decay [75]. These include: (a) dilaton stabilization through a semi-perturbative unification, (b) coupling unification with a better prediction for $\alpha_3(m_Z)$ compared to that for MSSM, (c) a simple understanding of the inter-family mass hierarchy, and (d) a possible explanation of a 2.7 σ anomaly in $(g-2)_\mu$. The vector like families with mass of order 1 TeV can of course be searched for at the LHC.

9 Concluding Remarks

The neutrinos seem to be as elusive as revealing. Simply by virtue of their tiny masses, they provide crucial information on the unification-scale, and even more important on the nature of the unification-symmetry. In particular, as argued in Secs. 4 and 6, (a) the magnitude of the superK-value of $\sqrt{\delta m_{23}^2}(\approx 1/20$ eV), (b) the b/τ mass-ratio, and (c) the need for baryogenesis via leptogenesis, together, provide clear support for: (i) the existence of the $SU(4)$-color symmetry in 4D above the GUT-scale which provides not only the RH neutrinos but also B-L as a local symmetry and a value for $m(\nu_{\text{Dirac}}^\tau)$; (ii) the familiar SUSY unification-scale which provides the scale of M_R; and (iii) the seesaw mechanism. *In turn this chain of arguments selects out the effective symmetry in 4D being either a string-derived $G(224)$ or $SO(10)$-symmetry, as opposed to the other alternatives like $SU(5)$ or flipped $SU(5)' \times U(1)$.*

It is furthermore remarkable that the tiny neutrino-masses also seem to hold the key to the origin of baryon excess and thus to our own origin!

In this talk, I have tried to highlight that the $G(224)/SO(10)$-framework as described here is capable of providing a *unified description* of a set of phenomena including: fermion masses, neutrino oscillations, CP and flavor violations as well as of baryogenesis via leptogenesis. This seems non-trivial.

The neutrinos have clearly played a central role in arriving at this unified description, first (a) by providing a clue to the nature of the unification-symmetry (as noted above), second (b) by confirming certain group-theoretic correlations between the quark and lepton sectors as regards their masses and mixings (cf. $m(\nu_\tau)_{\text{Dirac}}$ versus m_{top} and $\theta_{\nu_\mu \nu_\tau}^{osc}$ versus V_{cb}), and (c) by yielding naturally the desired magnitude for the baryon excess. Hence the title of the paper.

The framework is also highly predictive and can be further tested by studies of CP and flavor violations in processes such as (a) $B_d \to \phi K_S$-decay, (b) $(B_S, \bar{B}_S)$-decays, (c) edm of neutron, and (d) leptonic flavor violations as in $\mu \to e\gamma$ and $\tau \to \mu\gamma$-decays, and in $\mu N \to eN$.

To conclude, the evidence in favor of supersymmetric grand unification, based on a string-derived $G(224)$-symmetry in 4D (as described in Sec. 3) or $SO(10)$-symmetry, appears to be strong. It includes:

- Quantum numbers of all members in a family,

- Quantization of electric charge,

- Gauge coupling unification,

- $m_b^0 \approx m_\tau^0$

- $\sqrt{\delta m^2(\nu_2 - \nu_3)} \approx 1/20$ eV,

- A maximal $\Theta_{23}^\nu \approx \pi/4$ with a minimal $V_{cb} \approx 0.04$, and

- Baryon Excess $Y_B \approx 10^{-10}$.

All of these features and more including (even) CP and flavor violations hang together neatly *within a single unified framework* based on a presumed string-derived four-dimensional $G(224)$ or $SO(10)$-symmetry, with supersymmetry. It is hard to believe that this neat fitting of all these pieces emerging as predictions of one and the same framework can be a mere coincidence. It thus seems pressing that dedicated searches be made for the two missing pieces of this picture-that is supersymmetry and proton decay. The search for supersymmetry at the LHC and a possible future NLC is eagerly awaited. That for proton decay will need a next-generation megaton-size underground detector.

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[31] If the effective non-renormalizable operator like $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H/M'$ is induced through exchange of states with GUT-scale masses involving renormalizable couplings, rather than through quantum gravity, M' would, however, be of order GUT-scale. In this case $\langle\mathbf{45}_H\rangle/M' \sim 1$, rather than 1/10.

[32] While $\mathbf{16}_H$ has a GUT-scale VEV along the SM singlet, it turns out that it can also have a VEV of EW scale along the "$\tilde{\nu}_L$" direction due to its mixing with $\mathbf{10}_H^d$, so that the H_d of MSSM is a mixture of $\mathbf{10}_H^d$ and $\mathbf{16}_H^d$. This turns out to be the origin of non-trivial CKM mixings (See Ref. [9]).

[33] The flavor charge(s) of $\mathbf{45}_H(\mathbf{16}_H)$ would get determined depending upon whether $p(q)$ is one or zero (see below).

[34] These effective non-renormalizable couplings can of course arise through exchange of (for example) $\mathbf{45}$ in the string tower, involving renormalizable $\mathbf{16}_i\mathbf{\bar{16}}_H\mathbf{45}$ couplings. In this case, one would expect $M \sim M_{\text{string}}$.

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